Damping Estimation from Engine Data with Varying Natural Frequencies

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ABSTRACT

Damping from engine running data may be required for a variety of reasons including model validation, health monitoring, and general troubleshooting activities. The advantages of determining damping under realistic operating loads and boundary conditions are well known as are some of the difficulties involved. One such difficulty in gas turbine applications is that there may be significant natural frequency changes due to thermal and centrifugal-stiffening effects. This paper describes a novel technique which compensates for natural frequency changes and which may be used for both engine order and asynchronous (random) data. The method uses a variable carrier frequency modulation technique to effectively stationarise the data. The method and application to a simulated gas turbine example is described herein. The method is however completely general and may be useful in other applications.

1.0 NOMENCLATURE

$w(t)$	angular frequency (function of time)
$w_n(t)$	angular natural frequency
$w_{n/e}(t)$	estimate of angular natural frequency
$w^T_{USB}(t)$	upper sideband frequency
$w_c(t)$	carrier frequency
w_s	constant frequency
Δ	constant
$w_r(t)$	instantaneous response frequency (rate of change of phase)
USB	upper side band

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LSB	lower side band
EO	engine order
$\theta(t)$	phase of carrier signal
$s(t)$	response signal
$s_{BP}(t)$	bandpass filtered response (around mode of interest)
$s_c(t)$	carrier signal
$s^T(USB,t)$	transformed response signal bandpass filtered to retrieve the USB components

superscript T implies variable associated with transformed signal; e.g. $w_r^T(USB,t)$ is the response frequency of the transformed response signal after bandpass filtering around the USB frequency components.

2.0 INTRODUCTION

Damping from engine running data may be required for a variety of reasons including model validation, health monitoring, and general troubleshooting activities. The advantages of determining damping under realistic operating loads and boundary conditions are well known as are some of the difficulties involved. One such difficulty in gas turbine applications is that there may be significant natural frequency changes due to thermal and centrifugal-stiffening effects.

When natural frequency changes occur, determination of damping factors are problematic whether one is considering the asynchronous random and continuous response or the synchronous and transient response of an Engine Order traversing a mode. This paper describes a new technique for analysing either data set to determine damping; the method is suitable for automation and compensates for frequency changes which may have a complex time profile. The engine running applications considered herein and which gave rise to this method have the additional difficulty that the excitation forces are unknown.

Consider random vibration responses first and the determination of unbiased and accurate spectra as the first step in the damping estimation process. The difficulty of measuring damping when the natural frequency is constant is well known, [1]. For a given level of damping, the mode will have a modal bandwidth which needs to be resolved in the spectral analysis; the frequency resolution is inversely proportional to the segment or block length and therefore the requirement to have high resolution implies long segment durations. However the need for accurate spectral estimates with low random errors implies using a large number averages. In most practical applications, the data duration is limited and a compromise needs to be found between the two requirements. When the natural frequency is changing, the normal frequency resolution and time averaging requirements cannot in general be met over the very short amount of data available in which the data may be considered to be stationary.

Engine order data is by its nature different to the random case. By definition engine order responses are caused by forces which have frequency components which are synchronous with the rotational speed of one of the engine shafts. The normal means of analysing this type of engine data is to conduct a speed up or down test and perform some form of order analysis to track through a specific order as it traverses the mode of interest. The tracked response profile may then be examined to determine the mode bandwidth or damping

(through a modal curve fit). For the case when the natural frequency is changing in the region of this engine order traversal, the response profile will be distorted and damping estimates will be in error.

The method of compensating for natural frequency changes described within this paper is appropriate when the response bandwidth considered is dominated by a single mode and the natural frequency profile is reasonably 'smooth'. The natural frequency profile requirement is very often met since the time constants associated with the underlying thermal and centrifugal effects is long relative to the modal vibration time constants. For the random excitation case the unmeasured force distribution is assumed to be approximately constant over the modal bandwidth of interest. Similarly, for the engine order case, the unmeasured generalised force associated with the order is assumed to be approximately constant over the mode traversal region being analysed.

3.0 METHOD

The method works by using a continuous time domain transformation to stationarise the data, [2]. Specifically, a carrier signal with time varying frequency (which is synchronous with the resonant frequency and mode being identified) is determined and used to modulate measured response data after it has first been narrowband filtered around the mode of interest. The varying carrier frequency is offset by a constant amount to ensure that the lower and upper sidebands of the transformed signal are well separated; the narrowband filtering of measurements ensures that signal components from modes outside the frequency range of interest do not mix with those in the primary range of interest. The variation of the resonant frequency is estimated from the Campbell plot and or model data and may be determined with reasonable accuracy because of the smooth functional form of this variation, [3]. The result of this process is an upper sideband which represents a stationarisation of the data to enable standard damping estimation techniques to be legitimately used.

The relevant transformation may be considered as follows. The angular carrier frequency is

$$w_c(t) = w_s - w_{n/e}(t) \tag{1}$$

The Campbell plot may be used to provide an estimate of the natural frequency variation $w_{n/e}(t)$; [3].

The carrier signal is

$$s_c(t) = \sin \theta(t) \tag{2}$$

where the carrier phase (θ) is determined via time integration of Equation (1). The required transformation is obtained by modulating the response signal which has been bandpass filtered around the mode of interest $\{s_{BP}(t)\}$ with the carrier signal. The resulting signal becomes

$$s(t) = s_{BP}(t) \,.\, s_c(t) \tag{3}$$

The modulation process creates sum and difference frequencies. Considering these so-called upper and lower sideband frequencies separately, any response frequency component $w_r(t)$ would be translated into the upper sideband so that

$$w^T_{USB}(t) = w_r(t) + \{w_s - w_{n/e}(t)\} \tag{4}$$

For engine order data, the tracked response data for the order and mode of interest is narrowband and has a frequency which is very close to the order excitation frequency. Also, assuming the natural frequency estimate is accurate, i.e. we have

$w_r(t) \approx w_{EO}(t)$ (5)

and

$w_{n/e}(t) \approx w_n(t)$ (6)

Substituting these approximations into equation (4) yields

$w^T_{USB}(t) \approx \{w_{EO}(t) - w_n(t)\} + w_s$ (7)

So that the upper sideband has an instantaneous frequency which is approximately equal to the excitation to natural frequency difference and translated by a constant user defined value. The tracked response profile as a function of this separation frequency may then be further analysed for the damping. The constant value enables the frequency component to be shifted to any desired part of the spectrum. In practice the upper sideband would be filtered out and subsequently processed as illustrated in the test case described below.

For random response data, the long-time averaged spectrum or power spectral density function is required in order to compute the damping and the transformation described above may also be used to stationarise the data. In this case, consider a response frequency which is separated from the natural frequency by Δ so that

$w_r(t) = w_n(t) + \Delta$ (8)

The response signal is again bandpass filtered around the mode of interest and modulated with the carrier signal defined in Equation (2). The resulting upper sideband components may then be filtered out. The instantaneous frequency component defined in (8) is now translated to

$w^T_{USB}(t) = w_r(t) + \{w_s - w_{n/e}(t)\}$ (9)

Substituting Equations (6) and (8) into (9) yields

$w^T_{USB}(t) = w_n(t) + \Delta + \{w_s - w_{n/e}(t)\}$

$\approx w_s + \Delta$ (10)

This is the required property since we see that time varying frequency components are mapped to a constant frequency to any desired part of the spectrum and that the separation frequency Δ is maintained and now between the user defined constant ws. Therefore a power spectral density estimate of the transformed signal may now be computed and, assuming the response is dominated by a single mode and that the excitation is spectrally uniform across a frequency range spanning the 'modal bandwidth regions', the mode bandwidth and hence damping may be estimated. This is illustrated in the following test case.

4.0 RANDOM DATA TEST CASE

The method may be demonstrated by simulating a simple time variant system subject to continuous random excitation. The figures show an example in which two time varying modes are responding to random excitation. The initial Campbell plot (Fig. 1) shows this time variation and the fact that a range of frequencies spanning the modal bandwidth are simultaneously and continuously being excited. The damping is related to the modal bandwidth which is small in comparison to the natural frequency changes; i.e. the ratio of modal

bandwidth to the change in the natural frequency is roughly 0.1 for this example, and standard techniques cannot correctly identify the bandwidth and hence damping. The second Campbell plot (Fig. 2) shows the result after the original time history is transformed in the manner described above. The highest frequency modal responses represent the upper sideband of the higher frequency mode which now appears stationary (i.e. horizontal fuzz indicating constant natural frequency). This part of the spectrum may now be analysed to yield the modal bandwidth and damping; e.g. by mode bandwidth estimation from the power spectrum which is shown in Figure 3. In this particular case the process yielded damping estimates within 10% of the model parameter value.

5.0 ENGINE ORDER TEST CASE

The engine order simulation is of a single time varying mode whose natural frequency varies linearly with time from 85Hz to 110Hz. The engine order excitation frequency varies linearly with time from 80Hz to 120Hz. The response profile reveals a peak response at approximately 93.7 Hz. The –3dB modal bandwidth in this case is approximately 0.94Hz for a constant Q system of 100. The transformation is then conducted using a constant frequency value of $w_s = 100\pi$ and modulating the response signal as described above; the spectrum of the resulting signal is shown in Fig. 4. In this case the upper sideband is translated to the lower part of the spectrum. (Note: The plots in this example have 'frequency' axes which are respectively in terms of frequency array data point number such that the frequency is equal to the data point number times the frequency resolution. Similarly for the time axes. This is simply a feature of the plotting programs used.) The transformed signal may now be bandpass filtered to give a response versus time profile; a plot of this profile normalised to give a unit peak response is shown in Fig. 5. The corresponding instantaneous frequency (determined from the rate of change of phase of the USB response time history) is plotted in Fig. 6. The –3dB modal bandwidth computed from the transformed data is found in this case to have a 3% difference from the model value. The corresponding response profile and frequency plots for the original response is respectively shown in Figures 7 & 8: The response profile shows an apparent modal bandwidth of 2.1 Hz which is approximately twice the actual bandwidth. The frequency profile of the response closely follows the engine order excitation demonstrating that the sweep rate is sufficiently slow to ensure the response is effectively steady state.

6.0 DISCUSSION & CONCLUSIONS

The capability to estimate damping under conditions when natural frequencies are changing is particularly useful when analysing engine running data from speed up/down tests. Apart from the obvious cases in which it provides a means when none exist, it will also reduce the significant uncertainties which exist even when frequency changes are subtle but nonetheless influential. The method described within this paper provides a means for compensating for natural frequency changes when considering either synchronous engine order data or asynchronous random data and has been demonstrated to work with simulated data. The primary assumptions are that the response is dominated by a single mode and that the excitation is spectrally uniform over the modal bandwidths. The method is well suited to automation which is important in data analysis environments requiring large data throughput rates.

7.0 ACKNOWLEDGMENTS

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8.0 REFERENCES

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[2] Kurt-Elli, H., A method for the processing of oscillatory data, Rolls-Royce plc patent application DY3296, 2004.

[3] Allwood, R.J., King, S.P., & Pitts, N.J., The automatic interpretation of vibration data from gas turbines, The Aeronautical Journal of the Royal Aeronautical Society, March 1996.

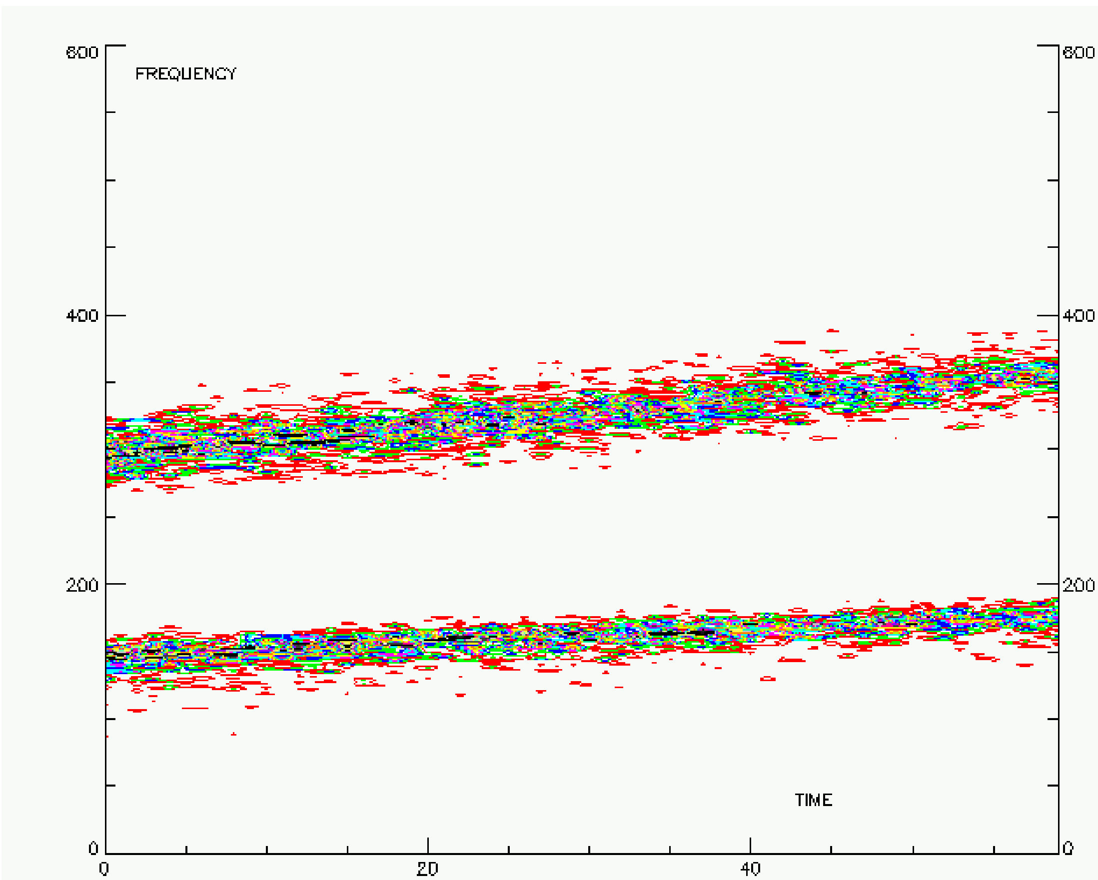

FIGURE 1: Campbell plot showing two time varying modes subject to random excitation.

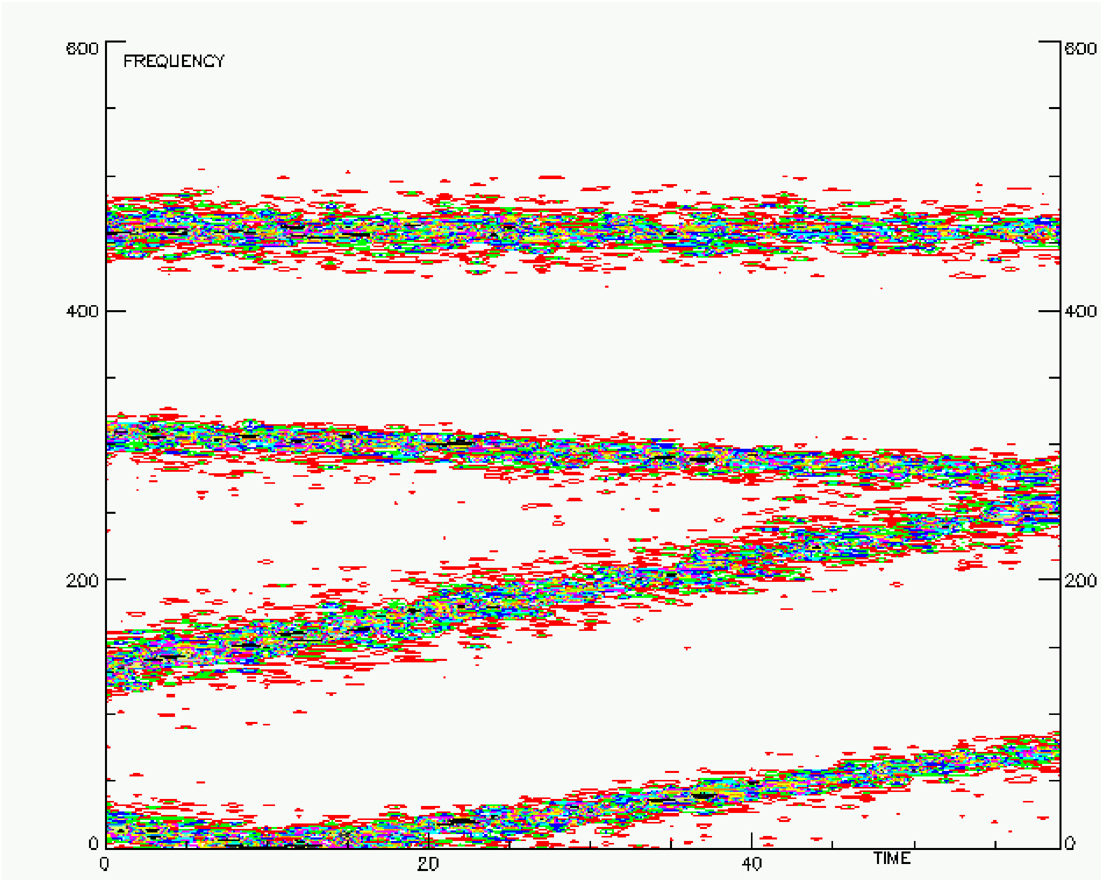

FIGURE 2: Campbell plot following continuous time domain transformation of data.

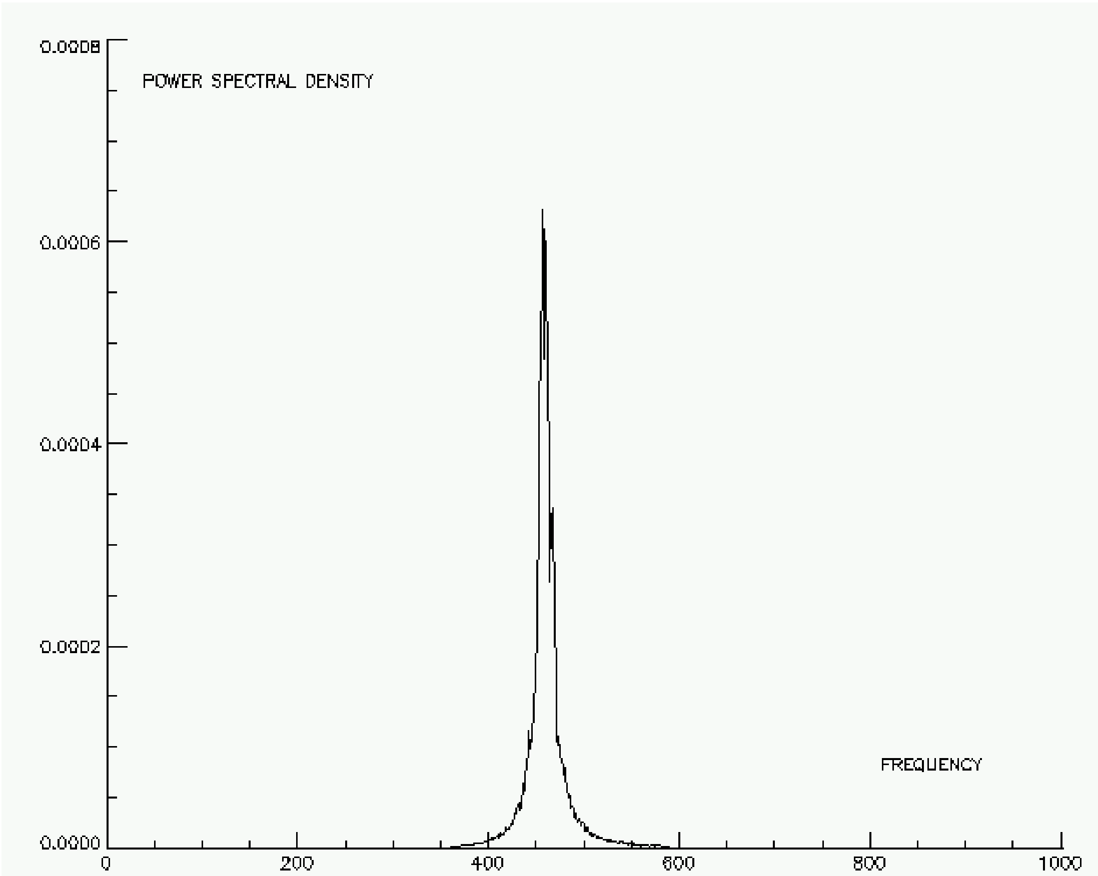

FIGURE 3: Power Spectral Density from which modal bandwidth is measured.

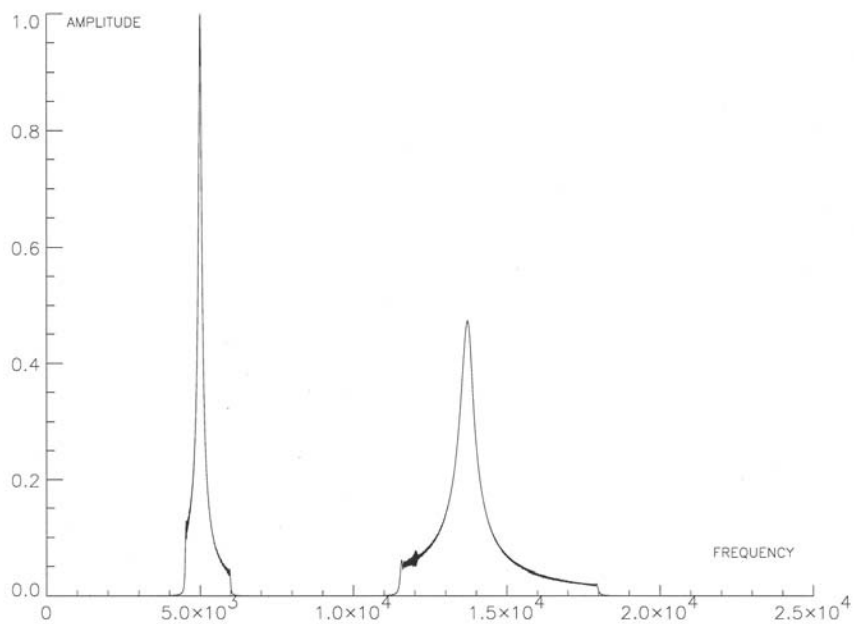

FIGURE 4 Spectral distribution of transformed response signal.

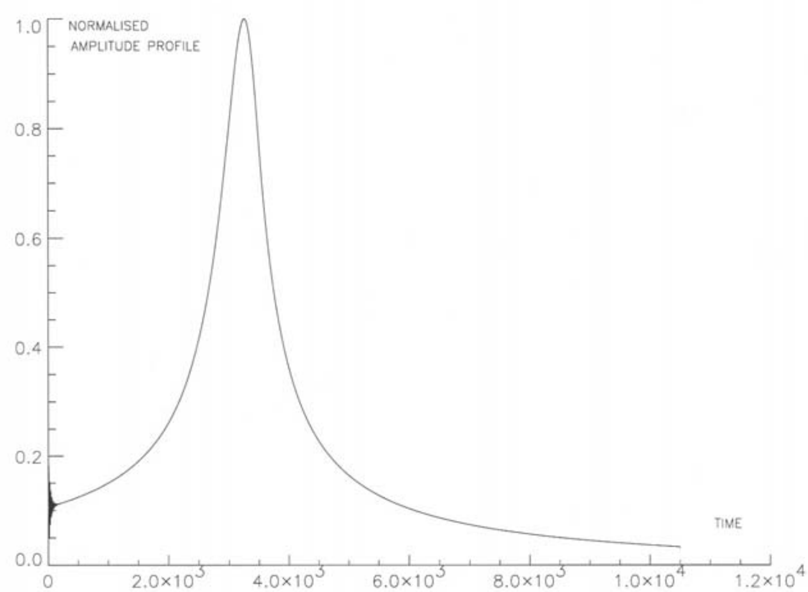

FIGURE 5 Normalised response profile (of filtered USB components) with respect to time.

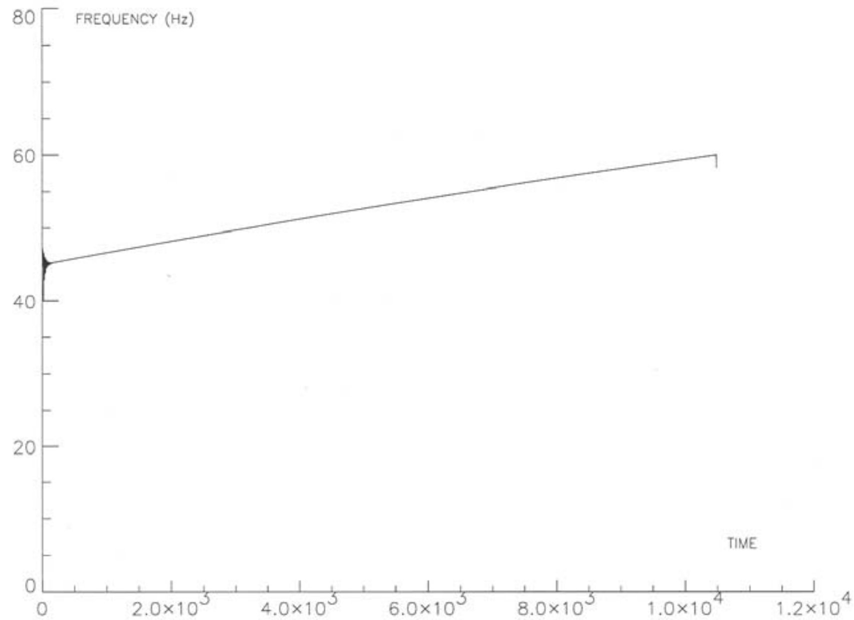

FIGURE 6 Instantaneous frequency of transformed response (after bandpass filtering for USB components).

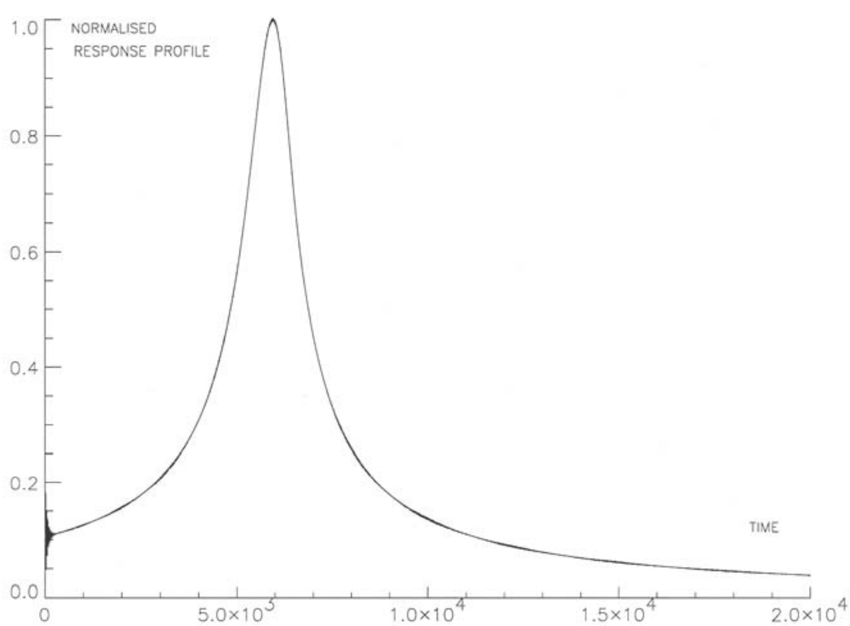

FIGURE 7 Response profile with respect to time.

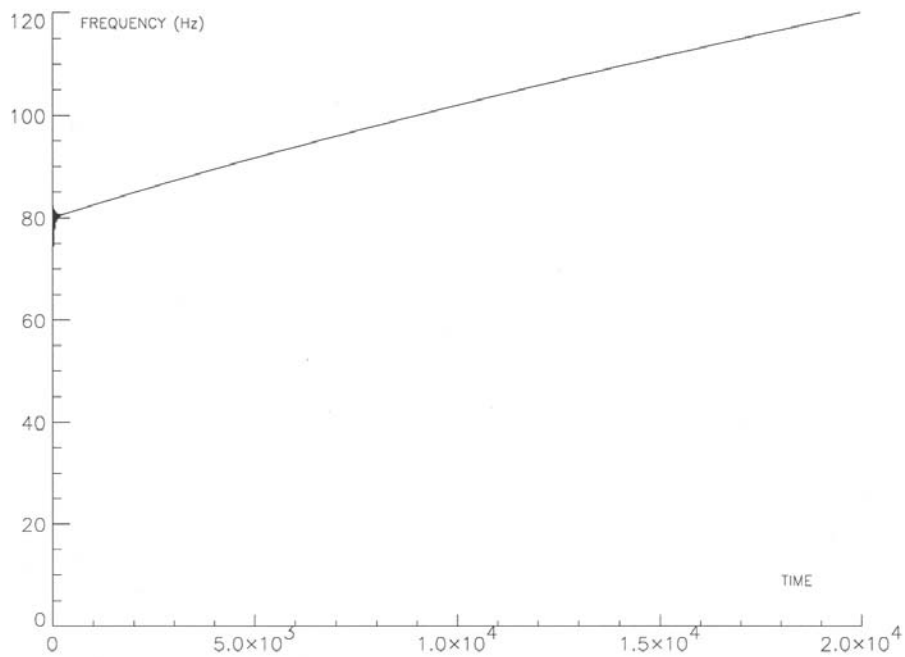

FIGURE 8 Instantaneous frequency of response with respect to time.

SYMPOSIA DISCUSSION – PAPER NO: 9

Author's name: H. Kurt-Elli

Discussor's name: Dr. J. Hou

Question: Is there a way to choose an optimal reference frequency, Ws, in order to minimise the relative error in damping estimation?

Answer: The issue of optimising Ws has not been investigated. However, experience to-date indicates that there appears to be little impact on the accuracy of damping estimates if Ws is chosen to ensure that mapped frequencies are not too different from the original frequency components, (e.g. 200 HZ mapped to 400 HZ but not to 2000 HZ).

Discussor's name: A. von Flotow

Question: The challenge in this approach is in the initial establishment of Wn(t) which must be done by an analyst to an accuracy of a small fraction of the band width, otherwise the method will over-predict damping. Do you agree?

Answer: Yes – broadly. The sensitivity of damping estimates to errors in the estimation of Wn(t) has not been investigated but it is true that the method requires a reasonably good estimation of the variation of the natural frequency. Bias errors in Wn/e(t) may be tolerated but random errors would need to be small relative to the mode bandwidth.